Concurrent Transaction Frame Logic
Formal Semantics for UML
Activity and Class Diagrams

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Abstract
We propose Concurrent Transaction Frame Logic (CTFL) as a language to provide formal semantics to UML activity and class diagrams. CTFL extends first-order Horn logic with object-oriented class hierarchy and object definition terms, and with three new logical connectives that declaratively capture temporal and concurrency constraints on updates and transactions. CTFL has coinciding, sound and refutation complete proof and model theories. CTFL allows using a single language to (1) formally describe the semantics of both activity and class diagrams, (2) verify UML models based on these two diagrams using theorem proving and (3) implement the model as an executable, object-oriented logic program.

\textit{Key words:} UML semantics, Object-oriented logic programming, Concurrent Transaction Frame Logic.

1 Introduction

The Unified Modeling Language (UML) \cite{16} provides an intuitive, visually clarifying standard notation for specifying and modeling computational systems. UML specifications and models are far more precise and less ambiguous

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than their natural language counterparts. They go a long way into facilitating communication between all the actors involved in the development of a system. However, the current UML standard is merely semi-formal, since its semantics is only defined in natural language rather than in some rigorous mathematical notation. This severely hinders the construction and use of automatic development tools for model verification, behavioral code generation and code testing in UML-based system engineering processes. To overcome this limitation, various proposals have recently been put forward to provide formal semantics to various UML diagrams [8], [5], [4], [19], [7], [1], [14], [28]. These proposals are very diverse in terms of the formal languages they use to describe UML diagrams and the development task automation functionalities that can be provided by tools relying on these languages. However, proposals covering Activity Diagrams (AD) share a common tendency to:

- focus only on activity and statechart diagrams, in isolation, outside of their structural context provided by Class Diagrams (CD) and other structural diagrams;
- provide only operational semantics, which are often seen as helpful in practice mainly to CASE tool developers, with axiomatic semantics better geared towards application designers and denotational semantics better geared towards language designers [9];
- rely on structurally impoverished imperative or functional formal languages that do not fit well the structure rich Object-Oriented (OO) paradigm used in most UML-based development processes;
- rely on low-level, and often quite arcane formal languages [24] that forces the analyst to get into minute algorithmic details, that ought to be abstracted until implementation, or entirely through the use of declarative programming [25];
- rely on a combination of several languages, typically one language to formalize the UML diagram structure, another one to formalize desired temporal properties, another one to implement CASE tools reasoning about models using these two formal notations, and often yet a different one to implement the system under development from the UML model.

As a result, a development team wishing to leverage these proposals to combine the intuitive visual clarity of UML with the rigor, robustness and CASE-tool automation of formal methods faces a steep learning curve as well as a significant development time overhead at the modeling stage. Given that time to market is the most critical factor in most real-life development projects, alternative approaches are needed to widen the applicability scope of formal, UML-based development.

In this paper, we propose such an alternative approach to provide formal semantics to UML models. It is based entirely on a non-monotonic variant of First-Order Horn Logic (FOHL). Although this approach has the potential
to provide semantics and CASE tools for the whole of UML, in this paper, we present a proposal focused on the formal semantics of an activity diagram contextualized by a class diagram.

We show how Concurrent Transaction Frame Logic (CTFL) [12] can provide formal semantics for both activity and class diagrams. CTFL is the straightforward integration of two orthogonal yet synergetic extensions of FOHL:

- *Frame Logic (FL)*, an object-oriented extension dealing with complex structural modeling with inheritance hierarchies,
- *Concurrent Transaction Logic (CTL)*, a non-monotonic extension dealing with complex behavioral modeling with concurrent logical database updates, transactions, process communication and temporal execution constraints.

Our approach is based on a mapping between the elements of UML activity and class diagrams and the constructors of CTFL. Through this mapping, these UML diagrams are given proof theoretical and model theoretical formal semantics: that of the CTFL program onto which they are mapped.

The rest of the paper is organized as follows. In section 2, we review the main elements of UML activity and class diagrams, illustrating each of them on a simple example model. In section 3, we review the object-oriented and non-monotonic constructs of CTFL illustrating them on the same example. In section 4, we provide a systematic mapping between the elements presented in section 2 and the constructs presented in section 3. This mapping defines our UML activity and class diagram formal semantics proposal. In section 5, we point out the main differences and advantages of our approach as compared to related work. In section 6 we review the contributions of the paper and outline directions for future work.

## 2 UML Class and Activity Diagrams

UML is a diagrammatic and textual language for specification and modeling in Object-Oriented Software Engineering (OOSE). In OOSE, the key software structure is the class. A class is an encapsulated, generic description of objects with similar structure, behavior, and relationships. An illustrative class diagram example is given in Figure 1. It is an extension of the Royal & Loyal (R&L) company information system class diagram presented in [29]. R&L manages fidelity programs for various companies, offering regular customers diverse bonuses such as air miles or discount points. A class diagram specifies the signature of each class, *i.e.*, the *attributes* used to represent the state of the objects of the class, together with constraints on their types, and the *methods*.

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5 We do not cover here the whole complexity of class diagrams, leaving this topic for a separate publication. Instead, we concentrate on the main features of class diagrams that are relevant to provide context to activity diagrams.
used to represent the behavior of these objects together with constraints on the type of their input parameters and return values. For example in Figure 1, the class `customer` models a fidelity program customer with attributes `name` and `title` of type `string`, a boolean attribute `isMale`, a `dateOfBirth` attribute of type `date`, and an `integer` returning method `age()`.

A class diagram also specifies the relationships between the defined classes. There are three main types of relationships: the `specialization` relationship to specify the hierarchy along which classes inherits attributes and methods, the `aggregation` and `composition` relationships to assemble complex objects from simpler ones viewed as parts, and the general purpose `association` for other relationships. These relationships can be labeled with cardinality constraints on the number of elements involved at each end of them. For example in Figure 1, `Earning` and `Burning` transactions are defined as subclasses of the general `Transaction` class, and each member of this class is associated to one member of the `CustomerCard` class.

What a class diagram does `not` specify is the behavior encapsulated in the
methods of the classes. UML provides various other diagrams to that effect. A State Diagram is essentially a graph that represents a state machine. Its use is recommended to specify the changes that occur in the attribute values of a single object as a result of invoking its methods and that of other objects. It specifies the conditions that trigger such change and the resulting, new values. In contrast, an activity diagram is essentially a flowchart, and its use is recommended to represent the state changes that occur in the attribute values of several objects that are involved in the implementation of a use-case [20]. Use-cases are requirement diagrams that divide the functionalities of a system into a set of distinct elementary usages. They describe the actors and purpose involved in each such usage. In strictly object-oriented development, each use-case must in the end be implemented by one method of some class. An activity diagram can also be used to describe the behavioral decomposition and control flow of complex methods implemented by way of invoking methods of objects from various other classes [26]. Although all UML diagrams are useful and complementary for complex system development, use-case, class and activity diagrams can be viewed as the minimal core of UML with which simple object-oriented systems can be specified and modeled. This is why we chose activity and class diagrams as the initial focus of our research on a simple and practical UML model formal semantics.

An illustrative activity diagram example is given in Figure 2. It models the realization of the burn method of the LoyaltyAccount class from the class diagram of Figure 1. This method itself realizes the use-case of the same name in the R&L system requirement document. An activity diagram is a graph where nodes are activities or control constructs and arcs represent transitions between them. Activities are decomposed into atomic action states than can be neither decomposed nor interrupted, and activity states than can be interrupted and further decomposed into sub-activities. Such decomposition can then be represented by another finer-grained activity diagram. A complex activity can thus be modeled by a hierarchy of activity diagrams, linked to one another through activity states. In addition to its name, an action state can also contain a specification of the operation that it executes. Such specification can be precisely written using the Object Constraint Language (OCL) a textual annotation language, part of the UML standard, that incorporates most basic constructs of logic and algorithms in an intuitive syntax [29]. In the activity diagram of Figure 2, the BurnServiceItem node is an example of activity state which behavior is specified by another activity diagram (shown in Figure 3). All the other nodes of this activity diagram are examples of either action states or control constructs. Activity states can also include entry and exit actions to be executed immediately before entering and immediately after leaving the state (respectively). The control constructs of an activity diagram are: (1) if/merge pairs, that represent conditional branching to mutually exclusive threads, (2) fork/join pairs, that represent concurrent threads, and (3) synch states, that represent inter-
thread synchronization constraints. For example in the diagram of Figure 2, the branching node below the IsGasOption action node models that either the action GetDesiredServiceItem or the action GetGasDiscount must immediately follow (depending on the user’s choice in that action in a given invocation of the burn method). In contrast, the fork node at the top of the diagram of Figure 3, models that the CheckStockOfServiceItem and CheckPointsAvailability actions must both always concurrently follow the execution of the GetDesiredServiceItem action. In the same diagram, the synch state below the CheckPointsAvailability action models that the execution of the action UpdateLoyaltyAccount that follows in the same thread must wait for the completion of CheckStockServiceItem in the other concurrent thread.
Transitions arcs can be labeled by an *event* whose occurrence triggers the transition from one state to the next or by a *guard*, i.e., a pre-condition that must be verified for the transition to occur. Both events and guards can be precisely modeled with OCL expressions.

An activity diagram can also include *object flows* that link it to a related class diagram. An object flow associates an action or activity state to a class. An incoming flow specifies the class of the objects that the action expects as input parameters. A simple outgoing flow specifies the class of the object returned as output by the action or activity. An outgoing flow with side effects specifies the class of the objects whose attributes are altered by the execution of the action or activity. Which attributes are altered and how can be precisely modeled with OCL expressions. For example in Figure 3, two object flows model that the *UpdateLoyaltyAccount* action takes objects of the classes *ServiceItem* and *LoyaltyAccount* as input parameters and a third one that models the parameter of class *LoyaltyAccount* has its *points* attribute altered by that action.
3 Concurrent Transaction Frame Logic (CTFL)

CTFL is the integration of FL and CTL, two orthogonal extensions of First-Order Horn Logic (FOHL), the subset of classical first-order logic where all formulas are in implicative normal form [21] with only one conclusion in each implication. A FOHL formula (also called a logic program) is thus a conjunction of implicitly universally quantified implications, each one either:

- A definite clause of the form $c \leftarrow p_1 \land \ldots \land p_n$, where $c$, $p_1$, $\ldots$, $p_n$ are positive literals;
- A fact of the form $c \leftarrow \text{true}$, where $c$ is a positive literal - usually abbreviated as $c \leftarrow$.

3.1 Frame Logic (FL)

FL extends first-order Horn logic with two new classes of object-oriented logical terms: class definition terms and object creation terms. A class definition term specifies the superclass of a class together with its proper attribute filler and method return type constraints, following the syntactic pattern:

$$\text{class} :: superclass[\ldots \text{attr}_i \text{typOp}_i \text{type}_i, \ldots,$$
$$\text{meth}_j(\ldots, \text{param}_k^j, \ldots) \text{typOp}_j \text{type}_j \ldots]$$

There are four typing operators in FL that instantiate the $\text{typOp}_n$ in the above pattern: $\ast \Rightarrow$, $\ast \Rightarrow \Rightarrow$, $\Rightarrow$ and $\Rightarrow \Rightarrow$. The presence or absence of the $\ast$ prefix distinguishes between inheritable and non-inheritable type constraints, whereas the $\Rightarrow$ and $\Rightarrow \Rightarrow$ suffixes indicates whether the attribute or method is single valued or set valued.

An object definition term creates a new object instance of a class and assigns its proper attribute and method return values, follow the syntactic pattern:

$$\text{object} : \text{class}[\ldots \text{attr}_i \text{assignOp}_i \text{value}_i, \ldots,$$
$$\text{meth}_j(\ldots, \text{param}_k^j, \ldots) \text{assignOp}_j \text{value}_j \ldots]$$

There are four value assignment operators, that instantiate the $\text{assignOp}_n$ in the above pattern: $\ast \rightarrow$, $\ast \Rightarrow \rightarrow$, $\rightarrow$ and $\Rightarrow \Rightarrow$. They follow the same prefix and suffix conventions than the typing operators. In FL, methods do not have bodies as in imperative object-oriented languages. A method is executed when its return result logical variable unifies with a value during theorem proving. The only difference between attributes and methods is thus that a
method can take parameters.

FL class definition and object creation terms are called F-Molecules. Logical variables can appear in any position inside these molecules: as object name, class name, attribute name, method name, attribute value, method value or method parameter. This freedom provides FL with a high-order syntax that allows for very concise meta-level specifications. However, there exists a simple, tractable mapping from any F-Molecule to a conjunction of First-Order Horn Logic literals, which guarantees that semantically, FL remains a first-order logic [30].

In order to illustrate FL more concretely, let us examine some facts of Figure 4 that shows the FL facts representing the R&L class diagram of Figure 1. Facts 3 and 10 define loyaltyAccount and transaction as top-level classes (in which the : superclass element of the pattern is simply omitted), while facts 11 and 12 define earning and burning as two sub-classes of transaction. These four facts also define the type signature constraints on the attributes of these classes, such as points* => integer in the loyaltyAccount class, and on their methods parameters and return value, such as getServiceOption(string) * – > {supermarket; fly; gas}. This last constraint illustrates the disjunctive value syntax of FL, used in this example to codify a UML enumeration type. These FL facts also define associations through attributes which types are constrained to other classes of the diagram, such as customerCard in the definition of the transaction class.

A pair of proof and model theories of FL is given [11]. The proof theory consists of one isaReflexivity axiom, three inference rules for first-order Horn logic with equality, resolution, factoring and paramodulation, and nine new inference rules covering the object-oriented semantics: isaTransitivity, isaAcyclicity, subclassInclusion, typeInheritance, inputRestriction, outputRestriction, scalarity, merging and elimination. The model theory consists of a Herbrand model over a F-Molecule universe. In the same paper, the two semantics are proven to be coinciding, sound and refutation-complete.

3.2 Concurrent Transaction Logic (CTL)

Sequential Transaction Logic (STL) extends first-order Horn logic with two new transactional connectives: n-ary serial conjunction $\otimes$, and n-ary serial disjunction $\oplus$. Concurrent Transaction Logic (CTL) further extends STL with three additional ones: n-ary concurrent conjunction $\mid$, n-ary concurrent disjunction $\triangledown$ and unary atomic modality $\diamond$. These three connectives allow representing in a purely declarative and logical way ordering and synchronization constraints on the execution order of logical proof steps. They provide declarative proof-theoretic and model-theoretic semantics to logic programs and database updates and transactions, as well as to multi-agent and inter-process communication protocols.
**Fact 1:** customer[name*=> string, title*=> string, isMale*=> void, 
dateOfBirth*=> date, cards*=> customerCard, 
programs*=> loyaltyProgram, memberships*=> membership, 
age()*=> integer] ←.

**Fact 2:** customerCard[valid *= void, validFrom *= date, 
goodThru *= date, color *= silver;gold, printedName *==> string, 
owner *==> customer, membership *==> membership, 
transactions *==> transaction, checkCard() *==> void] ←.

**Fact 3:** loyaltyAccount[points*=> integer, membership*==> membership, 
transactions*==> transaction, earn(integer)*=> void, 
burn(integer)*=> void, isEmpty()*==> void, getPoints()*==> integer, 
updateLoyaltyAccount(integer)*=> void, 
cancelledBurning()*=> void, completedBurning(integer)*=> void, 
checkPointsAvailability(integer, integer)*=> void, 
getDesiredServiceItem(string)*=> serviceItem, getGasDiscount()*==> void, 
getServiceOption(string)*=> {supermarket;fly;gas}] ←.

**Fact 4:** loyaltyProgram[customers*=> customer, memberships*=> membership, 
serviceLevel(integer)*=> serviceLevel, partners*=> programPartner, 
enroll(customer)*=> void, checkEnrolled(customer)*=> void] ←.

**Fact 5:** membership[loyaltyAccount*=> loyaltyAccount, 
actualLevel*=> serviceLevel, card*=> customerCard, 
program*=> loyaltyProgram, customer*=> customer] ←.

**Fact 6:** programPartner[numberOfCustomers*=> integer, 
loyaltyPrograms*=> loyaltyProgram, deliveredServices*=> service] ←.

**Fact 7:** service[condition*=> void, pointsEarned*=> integer, 
pointsBurned*=> integer, description*==> string, 
programPartner*=> programPartner, serviceLevel*==> serviceLevel, 
serviceItem*==> serviceItem, transactions*==> transaction] ←.

**Fact 8:** serviceLevel[name*=> string, loyaltyProgram*=> loyaltyProgram, 
membership*==> membership, availableServices *==> service] ←.

**Fact 9:** serviceItem[option*=> {supermarket;fly;gas}, points*=> integer, 
name*=> string, quantity*==> integer] ←.

**Fact 10:** transaction[points*==> integer, date*==> date, 
status*==> {inProgress; cancelled; completed}, card*==> customerCard, 
loyaltyAccount*==> loyaltyAccount, 
program*==> transaction, program()*==> loyaltyProgram] ←.

**Fact 11:** burning::transaction[]←.

**Fact 12:** earning::transaction[]←.

Fig. 4. CTFL fact base giving formal semantics and implementing CD of Figure 1

The semantics of these new connectives is based on the logic programming concept of **execution as proof attempt**:

- The semantics of a serial conjunction $p \otimes q$ is: **first** execute $p$; **then,** if the execution of $p$ succeeded *i.e., if it was proven true*, execute $q$; if either of the two executions failed, so does $p \otimes q$; if they both succeeded, then does $p \otimes q$. 

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The semantics of a serial disjunction $p \oplus q$ is the negation of $p \oplus q$: first execute $p$, then irrespective of the result, execute $q$; if either of the two execution succeeded, so does $p \oplus q$; if they both failed, so does $p \oplus q$.

The semantics of a concurrent conjunction $p | q$ is: concurrently execute both $p$ and $q$. If either of the two executions failed, so does $p | q$. If they both succeeded, so does $p | q$.

The semantics of a concurrent disjunction $p \vartheta q$ is the negation of $p \mid q$: concurrently execute both $p$ and $q$; if either of the two execution succeeded, so does $p \vartheta q$; if they both failed, so does $p \vartheta q$.

In this context, the semantics of classical conjunction $p \land q$ becomes: execute both $p$ and $q$, either sequentially or concurrently in any order; if both succeeded, so does $p \land q$; if either one failed, so does $p \land q$.

Similarly, the semantics of classical disjunction $p \lor q$ remains the negation of the classical conjunction $p \land q$: execute both $p$ and $q$, either sequentially or concurrently in any order; if both failed, so does $p \lor q$; if either one succeeded, so does $p \lor q$.

The truth tables of these sequential and concurrent conjunctions (respectively disjunctions) are identical to that of classical conjunction (respectively disjunction). The difference between these new connectives and their classical counterpart lies only in their execution order constraints: specified and sequential for $\otimes$ and $\oplus$, concurrent for $\mid$ and $\vartheta$, and unspecified for $\land$ and $\lor$. Thus, whereas $\mid$, $\vartheta$, $\land$ and $\lor$ are commutative, $\otimes$ and $\oplus$ are not.

The atomic modality connective $\odot$, prevents the formula within its scope to be partially executed. If one element of an atomic conjunction scoped by $\odot$ fails, or if its execution is interrupted by some event, the other elements must be rolled back and all the objects that had been changed must be restored to their states prior to the start of the atomic conjunction execution. For example, if $q$ fails in the formula $\odot(p \otimes q)$, then all the state changes resulting from the execution of $p$ must be rolled back.

A key characteristic of TL is its deliberate focus on defining complex actions and transactions out of simpler ones. It does not include any atomic change nor synchronization primitives in itself. To be used in practice, it must thus be parameterized with a set of such primitives. Atomic change primitives useful for our purpose are insertion and deletion of logical facts in a logical database. Synchronization primitives useful for our purpose are sending and receiving synchronization messages across channels shared by several threads. Thus, $\text{CTL}\{\text{insert}(\text{Fact}), \text{delete}(\text{Fact}), \text{send}(\text{Channel,Fact}), \text{receive}(\text{Channel,Fact})\}$ provides a fully declarative formal semantics for non-monotonic first-order Horn logic and database with concurrent updates and transactions.

A pair of coinciding, sound and refutation complete proof and model theories of STL are given in [2]. Their respective extensions to CTL are given in [3]. The model theory is based on a multi-path structure that captures the possible states that a logical database can pass through when complex
transactions are applied to it. These transactions use the classical and transactional connectives of CTL to combine primitive updates. The proof-theory relies on one axiom that states database invariance through the application of the empty transaction, together with four inference rules for transaction definition application, database query, database primitive update and atomic transaction execution.

3.3 Integrating Frame Logic with Transaction Logic

Given that FL extends first-order Horn logic by introducing new terms and STL and CTL extends it by introducing new connectives, these extensions are orthogonal and can be straightforwardly combined, respectively yielding STFL and CTFL. To be precise, it is CTFL\{insert(Fact), delete(Fact), send(Channel, Fact), receive(Channel, Fact)\} that we propose as a formal language for UML activity and class diagram semantics.

While there is no currently available compiler for CTFL, execution platforms are available for two of its subsets: (1) Flora \[30\], compiles and efficiently executes STFL programs, and (2) CTR\(^6\) interprets CTL programs. Both these platform are implemented as layers on top of the tabled deductive engine XSB \[22\], a variant of Prolog that relies on an alternative resolution-based First-Order Horn Logic theorem proving procedure called SLG. This procedure makes XSB both far more declarative and efficient than standard Prolog. It implements the well-founded semantics \[27\] for negation and it caches partial proof results to avoid both the inefficient redundant computation and the left-recursion termination problems of standard Prolog.

To visually summarize the relationships among the CTFL formalisms and tools and the UML activity diagram and class diagram, we give a UML metamodel \[16\] of our approach in Figure 5.

4 Mapping a UML class diagram and set of activity diagrams to a CTFL program

In this section, we present our mapping of UML activity diagram and class diagram elements to the CTFL constructs. A CTFL class definition fact base gives the semantics of the class diagram. A CTFL rule base gives the semantics of the activity diagram. In what follows, we associate a generic pattern of each main class diagram and activity diagram element with the corresponding CTFL construct pattern that defines its formal semantics in our proposal.

\(^6\) http://www.cs.toronto.edu/~bonner/
4.1 Mapping a UML class diagram to a CFTL class definition fact base

(1) Class mapping

A UML class signature is mapped directly onto a FL class definition term as shown in Figure 6. Attribute and methods with Boolean values in UML do not possess any return value in FL. This is because, every FL attribute or method implicitly has two results: its logical truth value, which is Boolean and its object identifier.

\[
\text{class}[\text{attribute}_1 \rightarrow \text{type}_1, \\
\text{method}_1(\text{par}_1 \rightarrow \text{Type}_1) \rightarrow \text{type}_1].
\]
(2) **Association mapping**

A UML association is mapped onto FL attributes of the associated classes, following the multiplicity constraints. For example, in Figure 7, class1 has a set valued attribute referencing classe2 and vice-versa.

![Association mapping diagram]

**Fig. 7. Association mapping**

(3) **Specialization mapping**

A UML specialization relationship is mapped onto a FL subclass "::" operator, as in the second F-Molecule of Figure 8. The default inheritability of UML attributes and methods is mapped onto the type constraint operators prefixed by * that captures such semantics in FL. Examples from the R&L case study of these three mappings above were discussed in section 3.1.

![Specialization mapping diagram]

**Fig. 8. Inheritance mapping**

### 4.2 Mapping a UML activity diagram to a CTFL Rule Base

Each path in an activity diagram is mapped onto a CTFL clause which conclusion corresponds to the overall activity modeled by the diagram. The nodes and transitions of each path are then mapped onto the premises of the corresponding CTFL clause following the rules below.
(1) Mapping action states

An action state A with no OCL constraint to further specify the behavior of this action is mapped onto an atomic premise that appears in each of the clauses that represent the activity diagram paths where A appears. This general mapping is shown in Figure 9. This is the case for example of the mapping from action state A7 in Figure 2 and the getGasDiscount premise of rule 4 in Figure 10. Such mapping is also carried out for actions with OCL constraints. However, in this case, an additional clause is added, with the action as conclusion and the OCL constraints as premises. This is the case for example of the mapping from action state A1 in Figure 2 to rule 8 in Figure 10.

(2) Mapping activity states

An activity state D is mapped onto one to four premises that appear in each of the clauses that represent the activity diagram paths where D appears. This general mapping is shown in Figure 11. In the simplest case, where the activity has neither entry condition, nor exit action nor interrupting event, the single premise consist only of the name activity D and its parameters. This is the case for example of the mapping from the A5 activity state in Figure 2 onto the burnServiceItem(SI, LA) premise in rule 5 and burnServiceItem(SI, LA) premise in rule 6 of Figure 10.

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7 Mapping a logical OCL expression to a CTFL premise is beyond the scope of this paper.
• **Rule 1**: Path $\langle A_0, A_8 \rangle$
  \[ \text{burnActivity} \leftarrow \circ (\neg \text{checkEnrolled}(C1:customer) \otimes \text{cancelBurning}). \]

• **Rule 2**: Path $\langle A_0, A_1, A_8 \rangle$
  \[ \text{burnActivity} \leftarrow \circ (\text{checkEnrolled}(C1:customer) \otimes \neg \text{checkCard}(CC:customerCard) \otimes \text{cancelBurning}). \]

• **Rule 3**: Path $\langle A_0, A_1, A_2, A_8 \rangle$
  \[ \text{burnActivity} \leftarrow \circ (\text{checkEnrolled}(C1:customer) \otimes \text{checkCard}(CC:customerCard) \otimes \neg \text{checkPoints}(LA:loyaltyAccount) \otimes \text{cancelBurning}). \]

• **Rule 4**: Path $\langle A_0, A_1, A_2, A_3, A_7 \rangle$
  \[ \text{burnActivity} \leftarrow \circ (\text{checkEnrolled}(C1:customer) \otimes \text{checkCard}(CC:customerCard) \otimes \text{checkPoints}(LA:loyaltyAccount) \otimes \text{isGasOption}(SI:serviceItem) \otimes \text{getGasDiscount}). \]

• **Rule 5**: Path $\langle A_0, A_1, A_2, A_3, A_4, A_5, A_8 \rangle$
  \[ \text{burnActivity} \leftarrow \circ (\text{checkEnrolled}(C1:customer) \otimes \text{checkCard}(CC:customerCard) \otimes \text{checkPoints}(LA:loyaltyAccount) \otimes \neg \text{isGasOption}(SI:serviceItem) \otimes \text{getDesiredServiceItem}(SI) \otimes \neg \text{burnServiceItem}(SI, LA) \otimes \text{cancelBurning}). \]

• **Rule 6**: Path $\langle A_0, A_1, A_2, A_3, A_4, A_5, A_6 \rangle$
  \[ \text{burnActivity} \leftarrow \circ (\text{checkEnrolled}(C1:customer) \otimes \text{checkCard}(CC:customerCard) \otimes \text{checkPoints}(LA:loyaltyAccount) \otimes \neg \text{isGasOption}(SI:serviceItem) \otimes \text{getDesiredServiceItem}(SI) \otimes \text{burnServiceItem}(SI, LA) \otimes \text{completeBurning}(SI)). \]

• **Rule 7**: state $A_0$
  \[ \text{checkEnrolled}(C1) \leftarrow C1:customer. \]

• **Rule 8**: action state $A_1$
  \[ \text{checkCard}(CC) \leftarrow CC:customerCard \land CC.valid. \]

• **Rule 9**: action state $A_2$
  \[ \text{checkPoints}(LA) \leftarrow LA:loyaltyAccount \land LA.points > 0. \]

• **Rule 10**: action state $A_3$
  \[ \text{isGasOption}(SI) \leftarrow SI:serviceItem \land SI[option=gas]. \]

• **Rule 11**: action state $A_6$
  \[ \text{completeBurning}(SI) \leftarrow SI:serviceItem \land \text{insert}(\text{burning}[points=SI.points, status=completed])). \]

• **Rule 12**: action state $A_8$
  \[ \text{cancelBurning} \leftarrow SI:serviceItem \land \circ (\text{insert}(\text{burning}[status=cancelled])). \]

Fig. 10. CTFL rule base giving formal semantics and implementing AD of Figure 2

Fig. 11. Activity state mapping

In the most complex case, with all the optional elements present, there is
one additional premise per element. Three are grouped in a serial conjunction in the following order: the entry action, then the complex activity D, and finally the exit action. These three premises are surrounded by an atomic transaction operator. This rolls back the side effects of the entry action and the complex activity D if an interruption occurs before the execution of the exit action. This complex transaction is conjoined with a possible interrupting event in a concurrent disjunction.

In all cases, the entire activity diagram mapping process is recursively re-applied onto the sub-activity diagram that further specifies the behavior D. This results in the introduction of new clauses in the CTFL program. This is the case for example of the recursive mapping of the activity diagram in Figure 3 onto the rules of Figure 12.

$$\text{Rule 1: activity state } A_5$$
$$\text{burnServiceItem}(SI, LA) \leftarrow (SI:\text{serviceItem} \land LA:\text{loyaltyAccount}) \odot$$
$$(\text{checkStockOfServiceItem}(SI) \odot \text{send}(ch1, done) \odot$$
$$\text{receive}(ch2, done) \odot \text{updateLoyaltyAccount}(SI, LA))$$
$$\odot \neg(\text{checkPointsAvailability}(SI, LA) \odot \text{receive}(ch1, done) \odot \text{cancelBurning}).$$

$$\text{Rule 2: activity state } A_5$$
$$\text{burnServiceItem}(SI, LA) \leftarrow (SI:\text{serviceItem} \land LA:\text{loyaltyAccount}) \odot$$
$$((\text{checkStockOfServiceItem}(SI) \odot \text{send}(ch1, done) \odot$$
$$\text{receive}(ch2, done) \odot \text{updateLoyaltyAccount}(SI, LA))$$
$$\odot (\text{checkPointsAvailability}(SI, LA)) \odot \text{receive}(ch1, done) \odot \text{send}(ch2, done) \odot \text{updateServiceItem}(SI, LA)).$$

$$\text{Rule 3: action state } A_9$$
$$\text{checkStockOfServiceItem}(SI) \leftarrow SI:\text{serviceItem} \land SI.\text{quantity} > 0.$$

$$\text{Rule 4: action state } A_{10}$$
$$\text{checkPointsAvailability}(SI, LA) \leftarrow SI:\text{serviceItem} \land LA:\text{loyaltyAccount} \land$$
$$LA.\text{points} > SI.\text{points}.$$

$$\text{Rule 5: action state } A_{11}$$
$$\text{updateLoyaltyAccount}(SI, LA) \leftarrow \odot(Pre = LA:\text{loyaltyAccount}.\text{points} \odot$$
$$\text{delete}(LA.\text{points}) \odot \text{insert}(LA[\text{points} \leftarrow (Pre - SI.\text{points})).$$

$$\text{Rule 6: action state } A_{12}$$
$$\text{updateServiceItem}(SI) \leftarrow \odot(Pre = SI:\text{serviceItem}.\text{quantity} \odot$$
$$\text{delete}(SI.\text{quantity}) \odot \text{insert}(SI[\text{quantity} \leftarrow (Pre - 1))].$$

Fig. 12. CTFL rule base giving formal semantics and implementing AD of Figure 3

(3) Mapping fork and join pairs

The concurrent nodes between a fork and a join in an activity diagram are mapped directly onto a CFTL concurrent conjunction. This concurrent conjunction is the central element of a serial conjunction that starts with the guard on the transition leading to the fork and ends with the guard out on
(4) Mapping fork and join pairs with synch states

In a UML activity diagram, synch state bars may be used within concurrent threads to represent synchronized states. These synch states are mapped to the synchronization primitives that parametrize CTFL in our formal semantics proposal. An incoming arc to such a synch state from another thread is mapped onto a receive($\text{Channel, done}$) premise in each of the CTFL clauses that represent the activity diagram paths where the synch state appears. An outgoing arc from such a synch state to another thread is mapped onto a send($\text{Channel, done}$) premise in each of the CTFL clauses that represent the activity diagram paths where the synch state appears. The $\text{Channel}$ parameter is used to identify the other thread from which the incoming arc is coming or to where the outgoing arc is going. These send and receive actions are joint in serial conjunctions with the other actions of the thread where the synch state occurs. This general mapping is shown in Figure 14.

An example of a bi-lateral synchronization between two threads is given in the activity diagram of Figure 3. One thread includes action states A9 and A12, while the other concurrent thread includes action states A10 and A11.
These two threads are synchronized by two arcs between four synch states, two in each thread. The arc that goes out from the top synch state in the A9-A12 thread to the top synch state in A10-A11 thread forces the execution of the branching test below latter to wait for the completion of A9 in addition to the completion of A10. It is mapped onto the synchronization predicate \texttt{send(ch1, done)} and \texttt{receive(ch1, done)} in the premise of CTFL rules 1 and 2 of Figure 12. Similarly, the arc that goes out from the bottom synch state in the A10-A11 thread to the bottom synch state in the A9-A12 thread forces the execution of A12 to wait for the completion of the branching node in the A10-A11 thread in addition to the completion of A9. It is mapped onto the synchronization predicate \texttt{send(ch2, done)} and \texttt{receive(ch2, done)} in the same two CTFL rules 1 and 2 of Figure 12.

(5) Mapping branching nodes

A UML activity diagram can contain two different kinds of branching nodes: (1) Boolean ones, with two outgoing transitions, one corresponding to the result of the previous activity being true and the other corresponding to the result of that activity being false, and (2) multiple choice ones, with at least three outgoing transitions, each one distinguished by a guard. A sub-branch leading from an activity A to an activity B through a Boolean branching node is mapped onto a serial conjunction in the premise of the clause that represent the activity diagram paths where the branching node appears. If the sub-branch includes the positive outgoing transition of the node, this serial conjunction starts with A. If it includes the negative outgoing transition, the serial conjunction starts with \(\neg A\). In both cases, the serial conjunction ends with B. This is the case for example of the mapping from the top Boolean branching node in the diagram of Figure 2, onto the \texttt{checkEnrolled(C1 : customer)} premise of the CTFL rule 2 in Figure 10 and the \(\neg \texttt{checkEnrolled(C1 : customer)}\) premise of the CTFL rule 1 in the same figure. A sub-branch leading from activity A to activity B through a multiple choice guarded outgoing transition is mapped onto a serial conjunction starting with A, followed by the guard and then by B. These three mappings are illustrated in Figure 15.
4.3 Mapping Object Flows

In UML, object flows link the behavioral activity diagrams to the structural class diagram. They are mapped onto predicates that link the behavioral CTL clauses with the structural FL clauses in CTFL.

(1) Mapping input parameter object flows

An action state input parameter specification object flows is mapped onto an CTFL F-Molecule (representing the object specification) that appears as argument in the CTFL predicate occurrences named after the action state. This general mapping is shown in Figure 16. It is the case for example of the mapping from the object flow that links object c1 with action state A0 in the diagram of Figure 2 onto the C1 : customer F-Molecule that appears as argument of the checkedEnrolled predicate occurrences in the CTFL clause of Figure 10.

(2) Mapping object creation object flows

An action state object creation object flow is mapped onto an additional atomic insert primitive database update premise with an anonymous object in each of the CTFL clauses that represent the activity diagram path where the action state appears. This general mapping is illustrated in Figure 17. This is the case for example of the mapping from the object flow outgoing from action state A6 in the diagram of Figure 2 onto the insert(., burning[pointsSI.points, statuscompleted]) premise of CTFL rule 11 in Figure 10.
An action state object attribute alteration object flow is mapped onto an atomic serial conjunction of two primitive database updates, one that deletes the old value of the attribute, followed by one that inserts the new value. The atomic modality operator guarantees that no intervening event can occur between the deletion and insertion. This general mapping is shown in Figure 18. This is the case for example of the mapping from the object flow outgoing from action state A11 in the diagram of Figure 3 onto the premises of CTFL rule 5 in Figure 12. Applying the general mappings defined in Figures 6-9, 11, and 13-18 above to the particular class diagram of Figure 1 and activity diagrams of Figures 2 and 3 yields the CTFL program shown in Figures 4, 10 and 12. This CTFL program represents both the formal semantics of the three UML diagram and their implementation as an executable object-oriented logic program.

In our mapping, we do not consider the following UML activity diagrams constructs: swimlanes, deferred events. Swimlanes correspond to organizational units in a business model that are used to organize responsibility for action and sub-activities. They do not impact the execution semantics. Deferring an event \( e \) can be simulated by using the guard \([e \text{ occurred}]\) [7].

5 Related Work

We encountered three main previous proposals to provide formal semantics to UML activity diagrams.

[4] proposed a semantics based on a mapping of activity diagram elements onto transition rules of a multiagent ASM, i.e., an Abstract State Machine with extensions for concurrency. An ASM is essentially a finite automaton where transitions are labeled with rules defining its preconditions and effects. ASM rules appear to capture the operational semantics of an activity diagram in a low-level language of imperative flavor.

[19] proposed a semantics based on mapping an activity diagram onto a Labelled Transition System (LTS) in two steps, through an intermediate representation called a Finite State Process (FSP). [7] proposed a semantics
also based on a mapping to an LTS in two steps, but through a different intermediate representation called an Activity Hypergraph. One advantage of these last two approaches is the availability of automatic model checkers that take as input an LTS model description, together with some temporal or modal logic description of execution ordering and timing constraints.

These previous proposals have in common to define only the operational semantics for activity diagrams. In addition, they do not cover object flows nor class diagrams, therefore providing semantics for activity diagrams in isolation from their structural context in a UML model. They thus seem more relevant for the use of activity diagrams in modeling purely procedural concurrent systems, than for their use in object-oriented software engineering.

Our proposal is different in two ways. First, it provides a model-theoretic and a coinciding proof-theoretic semantics for activity diagrams, based on a non-monotonic extension of first-order Horn logic. As pointed out in [9], semantics based on such logic unify the flavor of denotational semantics brought about by the model theory with those of both axiomatic and operational semantics brought about by the proof theory. Second, our proposal provides a formal semantics for both activity and class diagrams, linked together through object flows, which makes it more geared towards object-oriented software engineering.

6 Conclusion

In this paper, we proposed to provide a formal semantics to UML activity and class diagrams by mapping their elements to constructors of CTFL, a non-monotonic, object-oriented extension of first-order Horn logic. Through this mapping, the semantics of the UML diagrams derives from the coinciding, sound and refutation-complete proof theory and model theory of CTFL. This semantics presents a number of advantages over previous proposals. Foremost, it makes possible to use a single language to:

(i) Formalize the structure of various UML diagrams;
(ii) Formalize desired temporal execution properties over them, simple ones directly in CTFL and arbitrary complex ones using an additional Event Calculus [23] layer that is straightforward to axiomatize on top of CTFL;
(iii) Verify their internal and cross-diagram consistency, completeness and temporal correctness through a combination of theorem proving and model checking;
(iv) Implement the verified model as executable code.

This multiple purpose, single language approach smoothens the learning curve of integrating formal methods with standard object-oriented development. It also brings into the same fold the fast prototyping convenience of the logic programming paradigm. UML is a high-level, declarative, object-oriented language. Representing its formal semantics in a language like CTFL that is
also high-level, declarative and object-oriented, rather than in a low-level, pro-
cedural, purely behavioral language - as in most previous approaches - greatly
simplifies automatic translation of UML diagrams into their formalization. In
addition, CTFL is both a formal specification language and a general purpose,
Turing-complete programming language. Consequently, the verified, formal
CTFL semantics of a UML model is already an implementation. This shuts
down the major loophole of dual language formal development, one for formal
specification, and a different one for implementation, namely that program-
mapping errors can easily be introduced during the implementation of a verified
model.

Our plans for future work are shown in Figure 19.

We are currently developing XSLT transformation rule bases to autom-
atically map a UML model consisting of class diagrams and sequential activity
diagrams onto an STFL program. One rule base will implement, in XML
syntax, the mappings presented in this paper starting from a standard XMI
format for these diagrams together with an XOCL format \[18\] for the OCL
constraints. The other one will simply convert the resulting STFL program
from the XTFL syntax onto the syntax accepted as input by the Flora-2 STFL
theorem prover. We intend to use this prover to verify the UML model. Veri-
ification will be implemented declaratively using STFL meta-level rules. In that
perspective, it is interesting to mention the XMC model checker \[17\], which,
like Flora-2, is also implemented on top of XSB. XMC verifies concurrent sys-
tems specified in a CCS-based \[15\] modeling language with respect to desired
temporal properties specified in the modal \(\mu\)-calculus \[13\]. The performance
of XMC has proven comparable on a set of benchmarks to procedural model
checkers such as SPIN [10] and Murphi [6].

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